Amplitude- and Phase-Based R-F Direction Finding Systems for Spacecraft Rendezvous and Formation Flying

George Purcell and Sien-Chong Wu
JPL Section 335

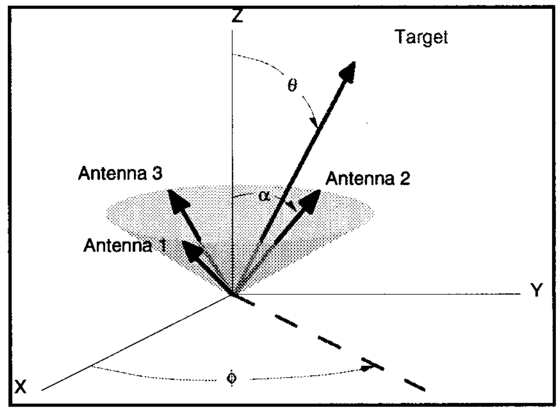
- Guiding Principle: Simplicity
- Amplitude-Based Direction Finding (DS4 Exploration)
  - Concept: 3 identical antennas pointed in different directions
  - Optimization of system parameters: beam size, cone angle
  - Effect of systematic errors: unmodeled azimuthal dependence of the beam pattern
  - Variants: more antennas, more beacons, rotating spacecraft
  - Summary: simple, inexpensive, compact, moderate accuracy ≈ 1/(voltage SNR)
- Phase-Based Direction Finding (ST3 Background)
  - Concept: 3 identical antennas and single-differenced phase observables
  - Effect of systematic errors: phase patterns of the antennas, phase offsets and drifts
  - Summary and comparison with amplitude-based approach

Concept for Amplitude-Based Direction Finding

➤ Beacon spacecraft transmits a monochromatic signal

➤ Lander:

- ✦ 3 identical antennas have simple beam shapes, gain decreasing monotonically from beam axis.
- ✦ Beam axes lie on a cone, half-angle a, at 120° intervals in azimuth.
- ✦ Beam angle of beacon signal, and therefore received signal amplitude, is different at each antenna.
- ✦ If beam shape is known, lander can determine direction to the beacon:
  ❶ 3 observables: A_1, A_2, A_3
  ❷ 3 estimated parameters: A_0, θ, ϕ

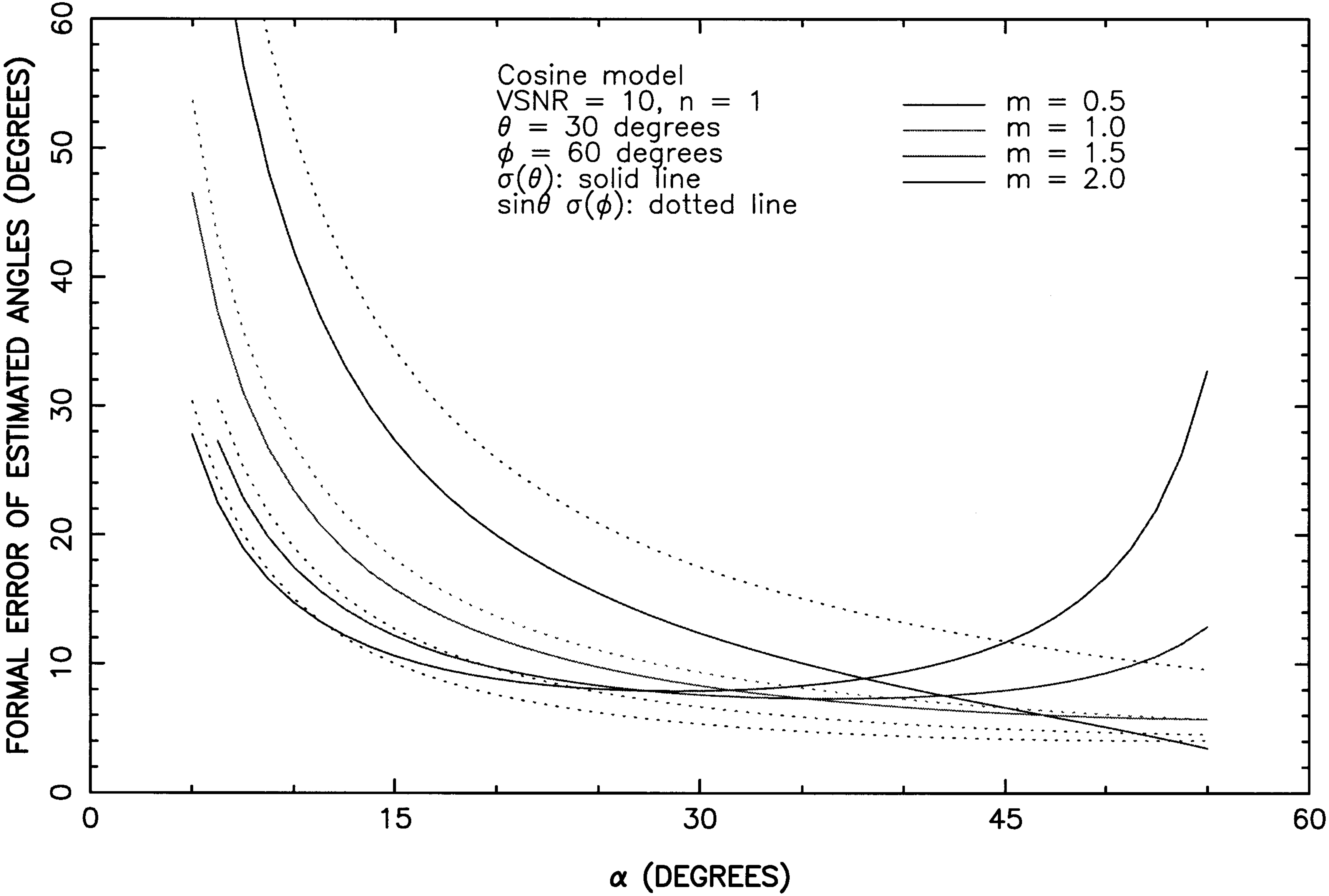


Geometry of Receiving Antennas

➤ Beam Models:

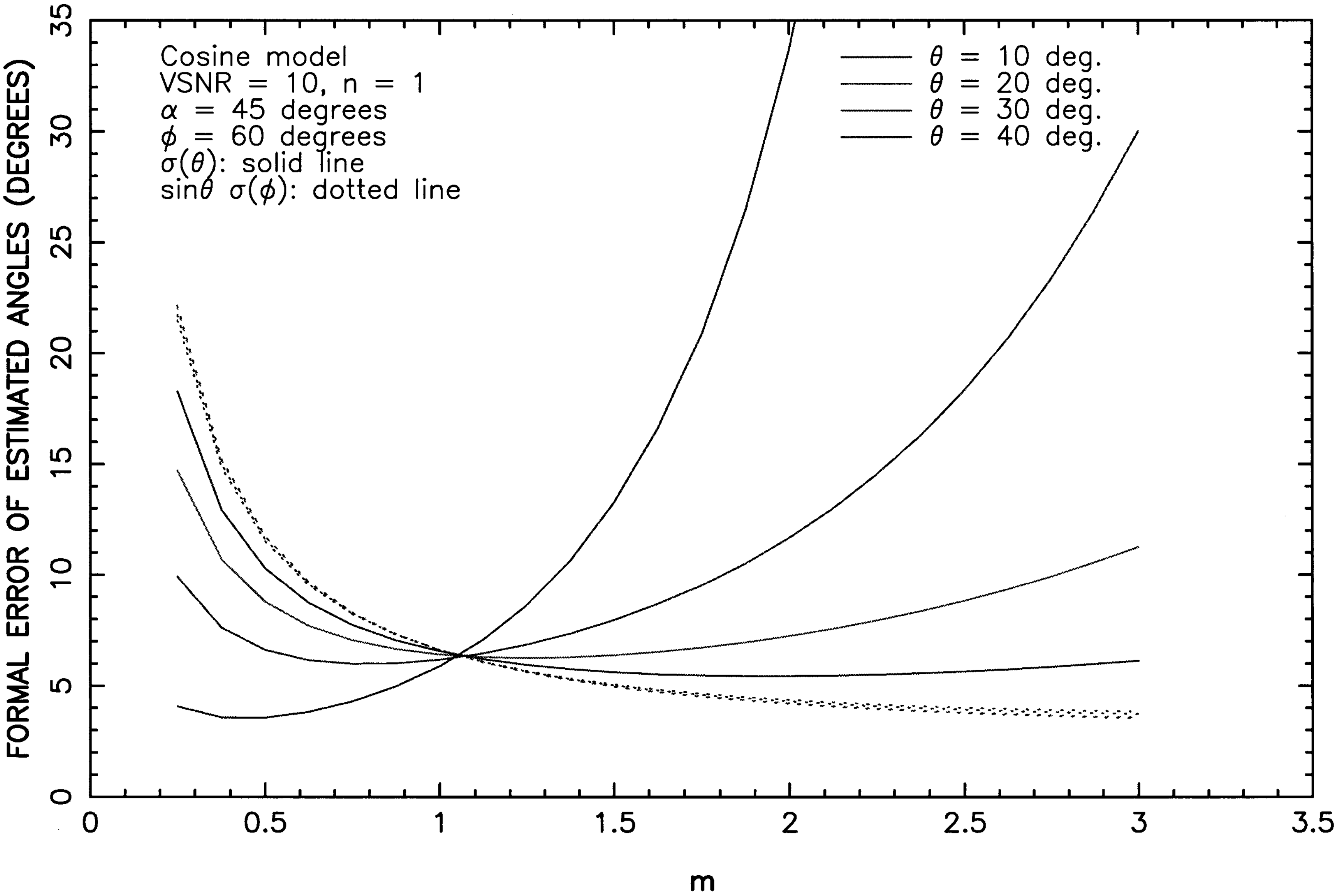
- ✦ Cosine: $A = A_0 \cos^m(n\gamma),\ 0 \le \gamma \le \pi/2n;\ A = 0,\ \pi/2n \le \gamma \le \pi$
- ✦ Gaussian: $A = A_0\ e^{-\gamma^2/\gamma_0^2},\ 0 \le \gamma \le \pi$
- ✦ Sinusoidal function of polynomial: $A = A_0 \sin[P(\gamma)]$, where P is a polynomial

DIRECTION FINDING UNCERTAINTY AS A FUNCTION OF ANTENNA CONE ANGLE, α



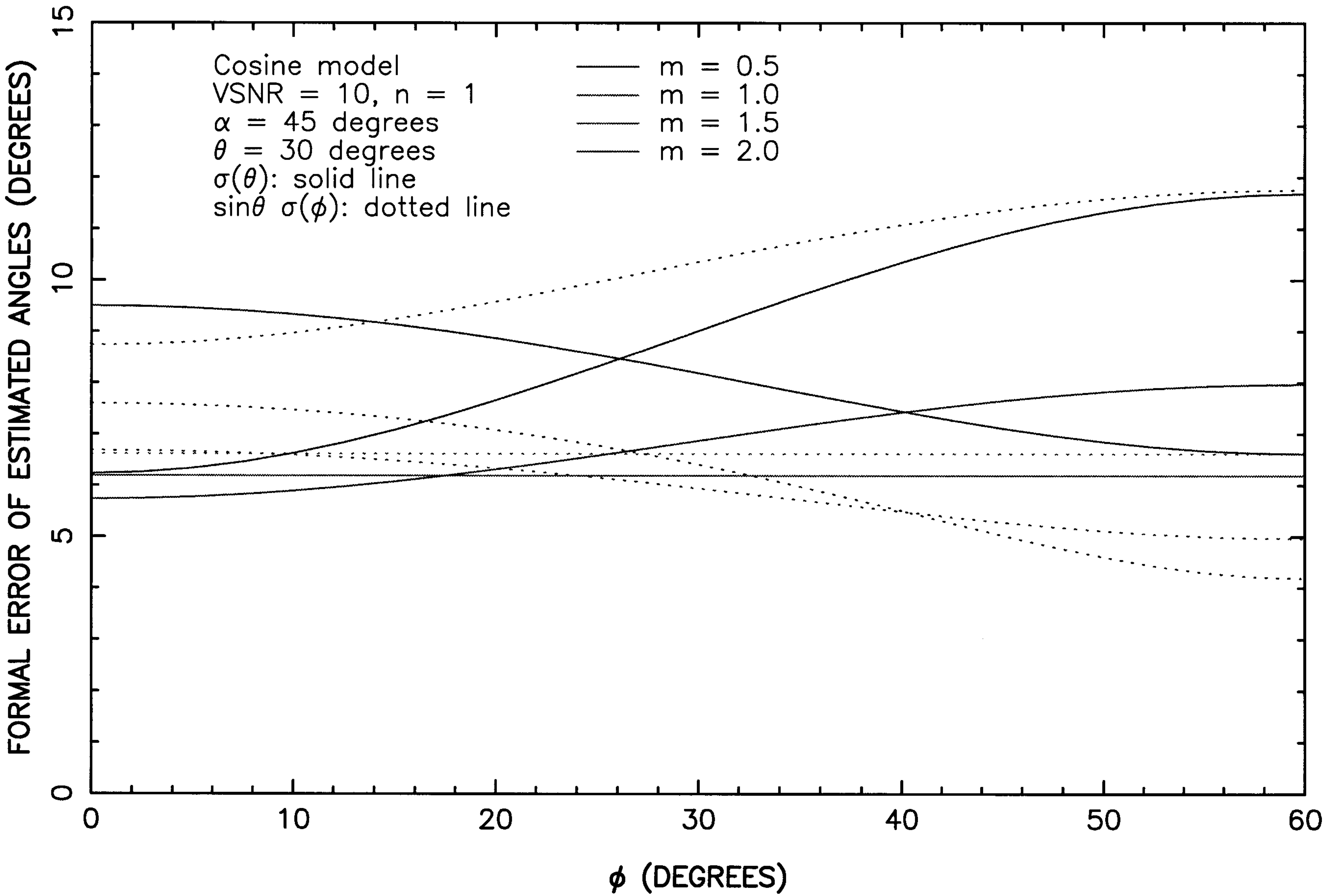
USER2:[GHP.AFF.DS-4]DESCANSO1.CRD;4 USER2:[GHP.AFF.DS-4]DESCANSO1.CPS;1 17-SEP-99 21:47:51 UT

DIRECTION FINDING UNCERTAINTY AS A FUNCTION OF ANTENNA BEAMWIDTH



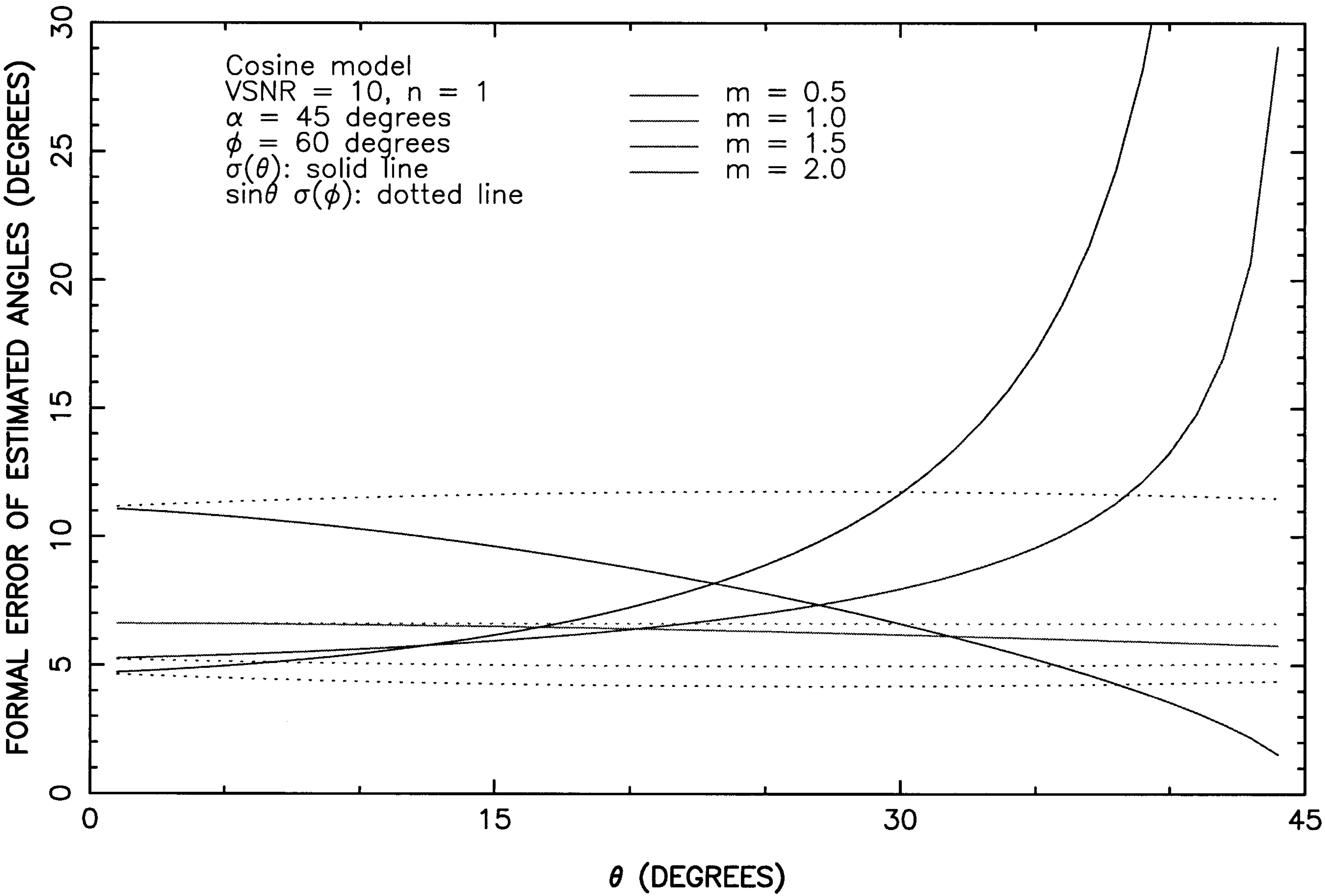
USER2:[GHP.AFF.DS-4]DESCANSO2.CRD;3 USER2:[GHP.AFF.DS-4]DESCANSO2.CPS;1 17-SEP-99 21:48:11 UT

DIRECTION FINDING UNCERTAINTY AS A FUNCTION OF TARGET AZIMUTH ANGLE, ϕ



USER2:[GHP.AFF.DS-4]DESCANSO3.CRO;3 USER2:[GHP.AFF.DS-4]DESCANSO3.CPS;1 17-SEP-99 21:48:26 UT

DIRECTION FINDING UNCERTAINTY AS A FUNCTION OF TARGET ZENITH ANGLE, θ



USER2:[GHP.AFF.DS-4]DESCANSO4.CRD;4 USER2:[GHP.AFF.DS-4]DESCANSO4.CPS;1 17-SEP-99 21:48:43 UT

Covariance Results for the Cosine Beam

- Cone angle parameter, α:
  - If the angle is too small, there is inadequate discrimination among the three antennas.
  - If the angle is too large, the beam angle at one or more antennas will be so large that the received power and beam-angle sensitivity will be small.
  - Optimum value: 35° to 45°.

- Beam concentration parameter, m:
  - If m is too small, beam-angle sensitivity is poor at small and moderate beam angles.
  - If m is too large, there is low power and beam-angle sensitivity at moderate and large beam angles.
  - Optimum value: ≈ 1.

- Because of symmetry, the uncertainty of the estimated direction of the beacon is periodic in ϕ, with a period of 120°.

- For good choices of the beam parameters, the uncertainties of the *formal* estimates of θ and $\phi \sin\theta$ are $\approx 1/\mathrm{SNR}_V$.

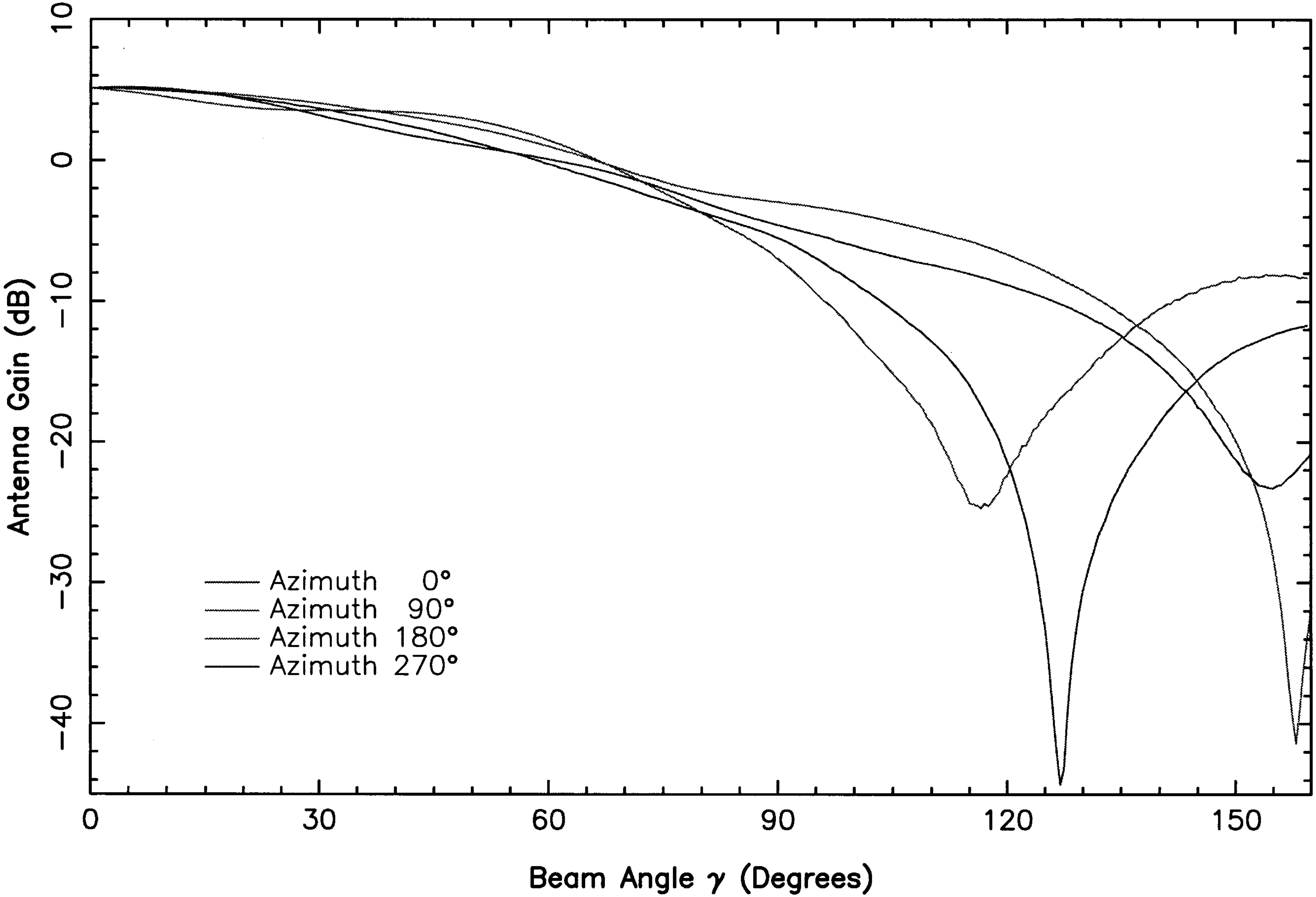
Effect of Errors in the Beam Model

- Probable dominant source of systematic error in direction-finding
- Test effect with the measured pattern of a Mars 01 Rover patch antenna
- Enhanced beam model with azimuthal dependence:

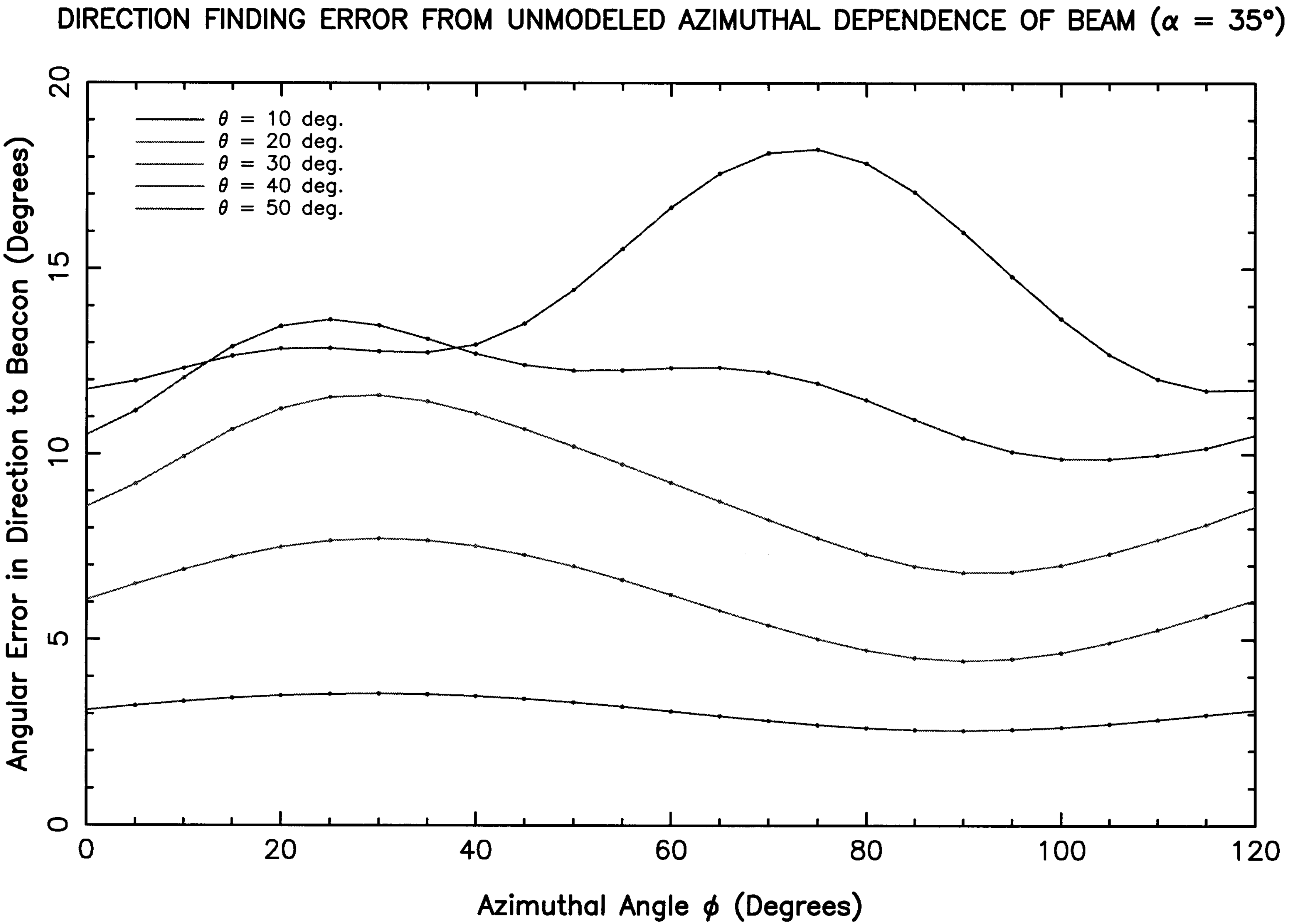
$$A(\gamma,\zeta) = A_0 \sin\left[P_0(\gamma)\right] g_0(\zeta) + A_{90} \sin\left[P_{90}(\gamma)\right] g_{90}(\zeta) + A_{180} \sin\left[P_{180}(\gamma)\right] g_{180}(\zeta) + A_{270} \sin\left[P_{270}(\gamma)\right] g_{270}(\zeta)$$

- Procedure:
  1. Given true beacon direction (θ_0, ϕ_0), find beam parameters γ and ζ at each antenna.
  2. Given γ and ζ, use the enhanced model to calculate the observables A_1, A_2, and A_3.
  3. Using nonlinear least-squares, solve for the estimated beacon direction (θ, ϕ), using the simple beam model without azimuthal dependence and starting from (θ_0, ϕ_0).
  4. Repeat to sample the full range of interesting beacon directions.
- Result: few-degree performance requires calibration at the few-percent level or better

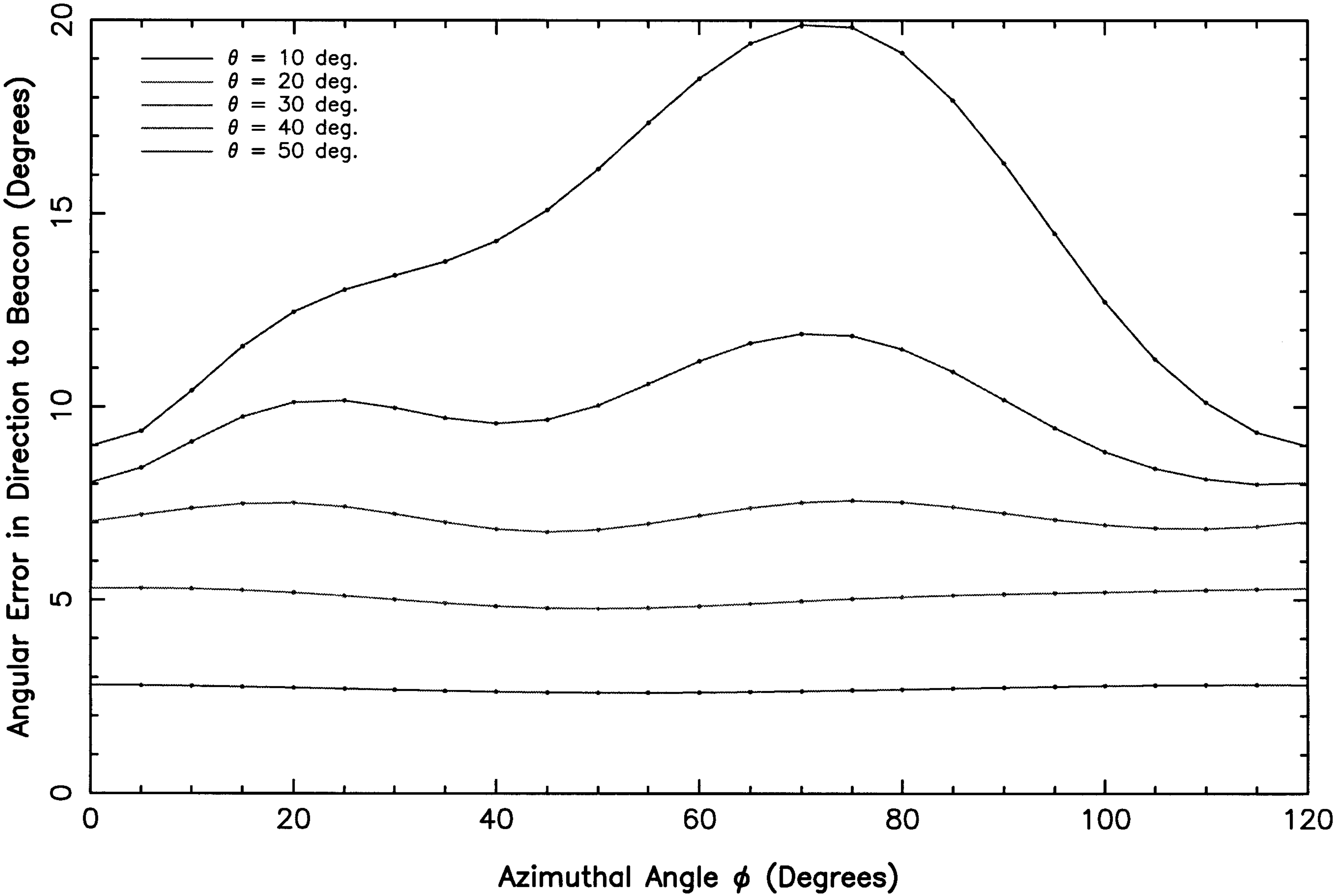
PATTERN OF MARS 01 PATCH ANTENNA FOR FOUR AZIMUTHS (NO MAST)



USER2:[GHP.AFF.DS-4]DESCANS05.CRD;3 USER2:[GHP.AFF.DS-4]DESCANS05.CPS;1 17-SEP-99 21:49:05 UT


DIRECTION FINDING ERROR FROM UNMODELED AZIMUTHAL DEPENDENCE OF BEAM (α = 35°)
θ = 10 deg.
θ = 20 deg.
θ = 30 deg.
θ = 40 deg.
θ = 50 deg.
Angular Error in Direction to Beacon (Degrees)
0
5
10
15
20
0
20
40
60
80
100
120
Azimuthal Angle ϕ (Degrees)

DIRECTION FINDING ERROR FROM UNMODELED AZIMUTHAL DEPENDENCE OF BEAM ($\alpha = 45°$)



USER2:[GHP.AFF.DS-4]DESCANSO7.CRD;2 USER2:[GHP.AFF.DS-4]DESCANSO7.CPS;1 17-SEP-99 21:49:30 UT

Summary & Remarks on Amplitude-Based Direction Finding

- Relatively Fool-Proof
  - R-F hardware for detection of a monochromatic signal is simple, small, lightweight.
  - Only calibrations required are antenna power patterns and cable losses.
  - Instrumental delays and phase shifts are irrelevant.
  - Placement of antennas is not crucial to the technique.
- Variants Possible
  - Use a single antenna on a rotating spacecraft.
  - Add antennas to detect signals from a larger solid angle.
  - Add beacons at different frequencies.
- Accuracy Limited to $\approx 1/SNR_V$ (radians).

Concept for Phase-Based Direction Finding

- ➤ Beacon transmits a monochromatic signal (same as for amplitude-based approach)

- ➤ Lander:
  - ✦ 3 identical antennas have simple, broad beam shapes.
  - ✦ Antennas lie at the vertices of an equilateral triangle with sides of length l, beam axes perpendicular to the plane of the antennas and parallel to the nominal line of sight to the beacon.
  - ✦ Absolute phase is not significant; relative phases specify the direction to the beacon.
  - ✦ Lander determines direction to the beacon:
    - ❶ Three observables, ϕ_1, ϕ_2, and ϕ_3, but only 2 are independent.
    - ❷ Two estimated parameters: θ, ϕ.

- ➤ Complications:
  - ✦ Phase calibration is more demanding than amplitude calibration (differential effects between antennas).
  - ✦ Phases are ambiguous in units of a cycle.

Covariance Results for Phase-Based Direction-Finding

- "Elevation" uncertainty: $\sigma(\theta) = \frac{\sqrt{2}\,\sigma_{\text{obs}}}{l}\sec(\theta) = \frac{\sqrt{2}\,\lambda_{\text{rf}}}{2\pi(SNR_V)l}\sec(\theta)$

- Azimuth uncertainty: $\sigma(\theta) = \frac{\sqrt{3}\,\sigma_{\text{obs}}}{l} = \frac{\sqrt{3}\,\lambda_{\text{rf}}}{2\pi(SNR_V)l}$

where σ_{obs} is the uncertainty of a phase measurement, in distance units, and
λ_{rf} is the r-f wavelength of the beacon signal.

- Formal error differs from that for an amplitude-based scheme by a factor of $\approx \lambda_{\text{rf}}/l$.

Summary of Phase-Based Direction-Finding and Comparison with an Amplitude-Based Approach

AMPLITUDE	PHASE
Requires power pattern(s) of the beams for calibration.	Requires phase pattern(s) of the beams for calibration.
Antenna placement is relatively unconstrained.	Antennas need to be set far apart ($l << \lambda$) to give this approach an advantage.
Observables are robust. Instrumental delay and phase shifts are irrelevant.	Observables are more vulnerable to corruption and integer-cycle errors.
Can be generalized to process multiple beacons.	Can also be generalized to process multiple beacons.
Formal error $\approx \dfrac{1}{SNR_V}$	Formal error $\approx \dfrac{\lambda_{rf}}{SNR_V\ l}$
Hybrid system could use amplitude measurements to initialize a phase solution.	